# Gravitinos in non-Ricci-flat backgrounds

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## Abstract

We discuss the gauge invariance and "mass" of the Rarita-Schwinger field in a background spacetime which is assumed to be Einstein but not necessarily Ricci-flat.

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Many elementary discussions of the spin 3/2 Rarita-Schwinger field begin by assuming that the spacetime background is Ricci-flat. This is a necessary condition for the existence of a reducible, massless RS field in four dimensions [1] (i.e. Ricci-flatness is forced on us unless we use the harmonic gauge $\gamma \cdot \psi = 0$). This special case is of limited interest, however, not least because many interesting supergravity backgrounds involve spaces of constant curvature, such as spheres and/or (Anti-)de-Sitter space.

In this letter we wish to clarify the issue of gravitino mass in a more general background. The Rarita-Schwinger field does not seem to couple consistently to non-gravitational external fields, and in the absence of self-interactions the equations of motion entail consistency conditions [1] which require the background to be Einstein; $R_{\mu\nu} = g_{\mu\nu} R/d$, where $d$ is the dimension of spacetime. So this is the most general possibility at the level of linearized supergravity, for example.

The most general form of the action contains two mass terms, but we will find that one of these can be shown by a change of field variables to be equivalent to a decoupled spin-half fermion. We show that the action is gauge invariant when the remaining mass term takes a value which is proportional to the square root of the Ricci scalar. For all mass values the Lagrangian can be written in a Dirac-like form.

With a Euclidean signature we start with the Lagrangian [2]

$$\mathcal{L} = \sqrt{g}(\bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho - m\bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu - \bar{m}\bar{\psi}^\mu \psi_\mu), \tag{1}$$

and make the decomposition

$$\psi_\mu = \varphi_\mu + \frac{D_\mu^T}{D \cdot D^T} D^T \cdot \psi + \frac{1}{d}\gamma_\mu \gamma \cdot \psi, \tag{2}$$

where $D_\mu^T = (\delta_\mu^\nu - \gamma_\mu \gamma^\nu/d) D_\nu$ is $\gamma$-transverse so that

$$\gamma \cdot \phi = D \cdot \phi = 0. \tag{3}$$

Then (1) becomes

$$\begin{aligned}\mathcal{L} = \sqrt{g}\Big[&\bar{\phi}^\mu (\not{D} + m)\phi_\mu + \bar{\psi} \cdot D^T \Big(\frac{\frac{d-2}{d}\not{D} - m + \bar{m}}{D \cdot D^T}\Big) D^T \cdot \psi - \frac{d-2}{d}\bar{\psi} \cdot D^T \gamma \cdot \psi \\ &- \frac{d-2}{d}\bar{\psi} \cdot \gamma D^T \cdot \psi + \bar{\psi} \cdot \gamma \Big(\frac{(d-1)(d-2)}{d^2}\not{D} + \frac{d-1}{d} m + \frac{1}{d}\bar{m}\Big)\gamma \cdot \psi\Big].\end{aligned} \tag{4}$$

Now if we put $\psi_1 = \sqrt{D \cdot D^T} D^T \cdot \psi$ and $\psi_2 = \gamma \cdot \psi$ then the change of variables $\psi_\mu \to (\phi_\mu, \psi_1, \psi_2)$ has a trivial Jacobian, and the coupled pair $(\psi_1, \psi_2)$ can be diagonalized from (4), using $D \cdot D^T = \frac{d-1}{d}\not{D}^2 - \frac{1}{4}R$. One of the resulting fields is a trivial auxiliary field; the other has the Lagrangian

$$\mathcal{L} = \sqrt{g}\bar{\psi}\Big(\frac{d-2}{d}\bar{m}\not{D} - \frac{d-1}{d}m^2 + \frac{d-2}{d}m\bar{m} + \frac{1}{d}\bar{m}^2 + \frac{(d-2)^2}{4d^2}R\Big)\psi. \tag{5}$$

If $\bar{m} \neq 0$ then we can normalize $\psi$ and the original Lagrangian (1) is therefore equivalent to

$$\mathcal{L} = \sqrt{g}[\bar{\phi}^\mu (\not{D} + m)\phi_\mu + \bar{\psi}(\not{D} + M)\psi], \quad M = \frac{\bar{m}}{d-2} + m - \frac{(d-1)m^2}{(d-2)\bar{m}} + \frac{(d-2)}{4d\bar{m}}R. \tag{6}$$

So the effect of having $\bar{m} \neq 0$ in (1) is the same as adding an additional spin-half fermion of mass $M$. Note that this conclusion is unaffected by the introduction of interaction terms which would merely couple $\psi$ to $\phi_\mu$. Thus we may without loss of generality set $\bar{m} = 0$.

Having done so, we conclude that $(\psi_1, \psi_2)$ produces *two* auxiliary fields, unless $M\bar{m} = 0$, in which case one of them decouples completely from the action, signalling the presence of a gauge invariance. Indeed, we can easily verify that for

$$m = \pm\frac{1}{2}(d-2)\sqrt{\frac{R}{d(d-1)}}, \tag{7}$$

(1) is invariant under

$$\delta\psi_\mu = D_\mu\lambda + \frac{m}{d-2}\gamma_\mu\lambda. \tag{8}$$

Using the methods of [3] it is straightforward to show that the Lagrangian in the "Feynman" gauge [4] can still be written in the Dirac form, with two Fadeev-Popov ghosts of mass $\pm\frac{1}{2}\sqrt{dR/(d-1)}$ and a "gauge-fixing" commuting spinor ghost of mass $5m/(d-2)$. In this case the field in the Lagrangian is unconstrained. Of course there is nothing to stop us making another choice of gauge, in which case the operator in the Lagrangian takes a slightly different form [5].

For the massive gravitinos we still have the constraints (3). By introducing Lagrange multiplier fields for these quantities we find that we can remove the constraints at the cost of introducing a pair of ghosts with masses $\pm\sqrt{m^2 + R/d}$. For the "massless" values (7) these coincide with the Fadeev-Popov ghosts. So for *all* values of the mass we can write the Lagrangian as[1]

$$\mathcal{L} = \sqrt{g}\bar{\phi}^\mu(\not{D} + m)\phi_\mu, \tag{9}$$

where $\phi$ satisfies (3). If $m$ satisfies (7) we have a single ghost of mass $5m/(d-2)$.

In conclusion, we have shown that the most general form of the Rarita-Schwinger Lagrangian with arbitrary dimension and mass can be written in the Dirac form (9). We identify the "massless" gravitino as the one of mass $m = \frac{1}{2}(d-2)\sqrt{R/d(d-1)}$, for which there is a single ghost of mass $5m/(d-2)$. There are also ghosts which are equivalent to imposing the constraints (3).

All this immediately allows many results involving anomalies, etc. [4,7] to be extended to the case of non-Ricci-flat spacetimes. It is also relevant to the study of supergravity compactifications beyond tree-level, as in [8], which is especially interesting in the context of the AdS/CFT correspondence [9].

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[1] Kaluza-Klein compactifications of supergravity [6] tend to yield gravitinos satisfying precisely the equations of motion implied by (9) and (3).

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